Rheology and Die Pressure

The extrudate is a fluid with flow properties that can be measured or estimated. Pressure to drive the flow through the die assembly is provided by the extruder. While the pressure needed for die flow is provided by the extruder, the pressure needed is a function of the volumetric flow rate, the rheology of the extrudate, and the dimensions of the flow path.

Because this is a fluid flow system, the system can be analyzed as sections of flow channels as common shapes. If a section of the die cannot be analyzed as a common shape, the concept of hydraulic diameter can be applied.

When designing a flow path, it can be helpful to calculate pressure drops in the system to ensure the extruder discharge pressure will not be excessively high or low. It can also be used to design dies that have similar discharge velocities across several shapes, if a die with multiple shapes is desired.

Excessively high discharge pressures may mean that the extruder is being operated beyond its design capabilities. High extruder discharge pressure may result in the extruder automatically shut down due to exceeding allowable pressure. Low discharge pressures could result in steam flashing in the die assembly for a puffed product, resulting in poor process control, or an inability to run the desired product. Multiple diameter circular openings may be used to create pellets for flakes with several different sizes, leading to a less “manufactured” look, or a more “natural” look. The land length in these different size openings may need to be defined to give the pellet lengths desired for each diameter of pellet.

Items to consider:

- Confirm the rheological model being used matches the rheological model used for the equations
  - Some reference use a different expression power law fluid flow.
- Keep the flow rate through the path correct, or calculate the resistance to flow in parallel for multiple paths.
  - If the flow rate is being split to 3 identical downstream paths, divide the volumetric flow rate by 3 and calculate for 1 path.
- The equations shown are for power law fluids, but can be used for Newtonian fluids by using a flow behavior index (*n*) of 1.

Definitions:

This page relates to a power law fluid model, see below (or https://en.wikipedia.org/wiki/Power-law_fluid) for the expression of the model used.

Power Law Fluid Model used:

$\tau = K\dot{\gamma}^n$ where τ is the shear stress, K is the flow consistency index, $\dot{\gamma}$ is the shear rate, and n is the flow behavior index.

Note that there are other expressions of the power law fluid model commonly used in plastics literature – make sure you understand the rheology model used for equations you are using, and correct for the expression of the power law fluid model as needed. The plastics reference books will often state how to convert to the power law fluid model as shown above.

Fluid flow equations related to common shapes:

Circle

Shear rate at the wall for a circle for a power law fluid:

$$\dot{\gamma}_w = \frac{\left(\frac{1}{n}+3\right)Q}{\pi R^3} = \frac{(1+3n)Q}{n\pi R^3}$$

Source: Conversion of the equation for a circle in Table 3.3 from Michaeli, W., 2003, Extrusion Dies for Plastics and Rubber, 3rd revised Edition, Hanser Gardner Pubications, Inc., Cincinnati, OH, USA. https://doi.org/10.3139/9783446401815.002

Volumetric Flow Rate for a circle for a power law fluid:

$$Q = \frac{n\pi R^3}{3n+1}\left(\frac{R\Delta P}{2LK}\right)^{\frac{1}{n}}$$

Source: Equation 12.26 from Levine, L. and Miller, R. C., Extrusion Processes. In Heldman, D. R., Lund, D. B., Sabliov, C. (Eds.), Handbook of Food Engineering, 2nd Edition, CRC Press, New York, NY, USA. https://doi.org/10.1201/9781420014372

Pressure drop for a circle for a power law fluid:

$$\Delta P = \frac{2KLQ^n}{R}\left[\frac{(3n+1)}{n\pi R^3}\right]^n$$

Source: Solved the volumetric flow rate equation for pressure drop.

Definition of Variables for the Circular Cross-section equations:

Variable	Description	Units (s = seconds, m = meters, Pa = Pascals)
$\dot{\gamma}_w$	Shear rate at the wall	$\frac{1}{s}$
n	Flow behavior index (shear thinning index)	Unitless
Q	Volumetric flow rate	$\frac{m^3}{s}$
R	Radius of the circular cross-section	m
ΔP	Pressure drop	Pa
L	Length of the flow path	m
K	Flow consistency index	$Pa \cdot s^n$

Cone

Shear rate at the wall for a cone for a power law fluid:

Use shear rate at the wall for a circular cross-section with the radius of interest

Volumetric flow rate for a cone for a power law fluid:

$$Q=\frac{\pi R_o^{\frac{1}{n}+3}}{(\frac{1}{n}+3)}\left[\frac{3n\Delta P\left(\frac{R_i}{R_o}-1\right)}{2KL\left(1-\left(\frac{R_o}{R_i}\right)^{3n}\right)}\right]^{\frac{1}{n}}$$

Source: Conversions of equation 3.60 using the the die conductance equation for a cone in Table 3.1, both from Michaeli, W., 2003, Extrusion Dies for Plastics and Rubber, 3rd revised Edition, Hanser Gardner Pubications, Inc., Cincinnati, OH, USA. https://doi.org/10.3139/9783446401815.002

Pressure drop for a cone for a power law fluid:

$$\Delta P=\left(\frac{Q(\frac{1}{n}+3)}{\pi R_o^{\frac{1}{n}+3}}\right)^n\left[\frac{2KL\left(1-\left(\frac{R_o}{R_i}\right)^{3n}\right)}{3n\left(\frac{R_i}{R_o}-1\right)}\right]$$

Source: Solved the volumetric flow rate equation for pressure drop.

Definition of variables for the cone equations:

Variable	Description	Units (s = seconds, m = meters, Pa = Pascals)
Q	Volumetric flow rate	$\frac{m^3}{s}$
R_o	Radius of the cone at the outlet end	m
R_i	Radius of the cone at the inlet end	m
n	Flow behavior index (shear thinning index)	Unitless
ΔP	Pressure drop	Pa
L	Length of the flow path	m
K	Flow consistency index	$Pa\cdot s^n$

Wide Slot

A wide slot is one where the width of the slot is much greater than the height of the slot.

Shear rate at the wall for a wide slot for a power law fluid:

$$\dot{\gamma}_w = \frac{2\left(\frac{1}{n}+2\right)Q}{W\,h^2} = \frac{2(1+2n)Q}{nW\,h^2}$$

Source: Conversion of the equation for a rectangular slit in Table 3.3 from Michaeli, W., 2003, Extrusion Dies for Plastics and Rubber, 3rd revised Edition, Hanser Gardner Pubications, Inc., Cincinnati, OH, USA. https://doi.org/10.3139/9783446401815.002

Volumetric flow rate for a wide slot for a power law fluid:

$$Q = \frac{nWh^2}{2(2n+1)}\left(\frac{h\Delta P}{2LK}\right)^{\frac{1}{n}}$$

Source: Equation 12.27 from Levine, L. and Miller, R. C., Extrusion Processes. In Heldman, D. R., Lund, D. B., Sabliov, C. (Eds.), Handbook of Food Engineering, 2nd Edition, CRC Press, New York, NY, USA. https://doi.org/10.1201/9781420014372

Pressure drop for a wide slot for a power law fluid:

$$\Delta P = \frac{2\,KLQ^n}{h}\left[\frac{2(2n+1)}{nWh^2}\right]^n$$

Correction for Narrow Slot

If the slot meets the constraint of:

$$\frac{W}{h} \leq 20$$

then apply the correction factor of:

$$F_p = 1.008 - 0.7474\left(\frac{h}{W}\right) + 0.1638\left(\frac{h}{W}\right)^2$$

Source: Equation 5.5 from Rao, N. S., and Schumacher, G., Design Formulas for Plastics Engineers, 2nd Edition, Hanser Gardener Publications, Cincinnati, OH, USA. https://doi.org/10.3139/9783446413009

in the pressure drop equation with the correction factor:

$$\Delta P = \frac{2KLQ^n}{hF_p}\left[\frac{2(2n+1)}{nWh^2}\right]^n$$

Source: Solved the volumetric flow rate equation for pressure drop.

Definitions of variables for the Wide Slot Cross-section equations:

Variable	Description	Units (s = seconds, m = meters, Pa = Pascals)
$\dot{\gamma}_w$	Shear rate at the wall	$\frac{1}{s}$
n	Flow behavior index (shear thinning index)	Unitless
W	Width of the slot (major dimension of the slot)	m
h	Height of the slot (minor dimension of the slot)	m
Q	Volumetric flow rate	$\frac{m^3}{s}$
ΔP	Pressure drop	Pa
L	Length of the flow path	m
K	Flow consistency index	$Pa \cdot s^n$
F_p	Correction factor for a narrow slot	Unitless

Thin Annulus

Shear rate at the wall for a thin annulus for a power law fluid:

$$\dot{\gamma}_w = \frac{2(\frac{1}{n}+2)Q}{\pi D h^2} = \frac{2(\frac{1}{n}+2)Q}{\pi(R_{ao}+R_{ai})(R_{ao}-R_{ai})^2}$$

Source: Conversion of the equation for an annular slit in Table 3.3 from Michaeli, W., 2003, Extrusion Dies for Plastics and Rubber, 3rd revised Edition, Hanser Gardner Pubications, Inc., Cincinnati, OH, USA. https://doi.org/10.3139/9783446401815.002

Volumetric Flow Rate for a thin annulus for a power law fluid:

$$Q = \frac{n\pi\bar{R}h^2}{2n+1}\left(\frac{h\Delta P}{2LK}\right)^{\frac{1}{n}}$$

Source: Equation 12.28 from Levine, L. and Miller, R. C., Extrusion Processes. In Heldman, D. R., Lund, D. B., Sabliov, C. (Eds.), Handbook of Food Engineering, 2nd Edition, CRC Press, New York, NY, USA. https://doi.org/10.1201/9781420014372

Pressure drop for a thin annulus for a power law fluid:

$$\Delta P = \frac{2KLQ^n}{h}\left[\frac{2n+1}{n\pi\bar{R}h^2}\right]^n$$

Source: Solved the volumetric flow rate equation for pressure drop.

Correction for Wide Annulus

If the slot meets the constraint of:

$$\frac{\pi(R_{ao}+R_{ai})}{R_{ao}-R_{ai}} \leq 20$$

then apply the correction factor of:

$$F_p = 1.008 - 0.7474\left(\frac{R_{ao}-R_{ai}}{\pi(R_{ao}-R_{ai})}\right) + 0.1638\left(\frac{R_{ao}-R_{ai}}{\pi(R_{ao}-R_{ai})}\right)^2$$

in the pressure drop equation with the correction factor:

$$\Delta P = \frac{2KLQ^n}{hF_p}\left[\frac{2n+1}{n\pi\bar{R}h^2}\right]^n$$

Definition of variables for the Thin Annulus Cross-section equations:

Variable	Description	Units (s = seconds, m = meters, Pa = Pascals)
$\dot{\gamma}_w$	Shear rate at the wall	$\frac{1}{s}$
n	Flow behavior index (shear thinning index)	Unitless
Q	Volumetric flow rate	$\frac{m^3}{s}$
D	Diameter of the center of the annulus (= (outer annulus diameter + inner annulus diameter)/2)	m
h	Height of the annulus (= (outer annulus radius – inner annulus radius)	m
R_{ao}	Outer annulus radius	m
R_{ai}	Inner annulus radius	m
$\bar{R}$	Radius of the center of the annulus (= (outer annulus radius + inner annulus radius)/2)	m
ΔP	Pressure drop	Pa
L	Length of the flow path	m

K	Flow consistency index	$Pa \cdot s^n$
F_p	Correction factor for a wide annulus	Unitless